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A closed string tachyon vacuum?

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Abstract: In bosonic closed string field theory the "tachyon potential" is a potential for the tachyon, the dilaton, and an infinite set of massive fields. Earlier computations of the potential did not include the dilaton and the critical point formed by the quadratic and cubic interactions was destroyed by the quartic tachyon term. We include the dilaton contributions to the potential and find that a critical point survives and appears to become more shallow. We are led to consider the existence of a closed string tachyon vacuum, a critical point with zero action that represents a state where space-time ceases to be dynamical. Some evidence for this interpretation is found from the study of the coupled metric-dilaton-tachyon effective field equations, which exhibit rolling solutions in which the dilaton runs to strong coupling and the Einstein metric undergoes collapse.

Keywords: Tachyon Condensation, Bosonic Strings, String Field Theory.

Contents

1. Introduction and summary

In the last few years the instabilities associated with open string tachyons have been studied extensively and have become reasonably well understood [1]. The instabilities associated with closed string tachyons have proven to be harder to understand. For the case of localized closed string tachyons — tachyons that live on subspaces of spacetime — there are now plausible conjectures for the associated instabilities and a fair amount of circumstantial evidence for them [2 – 6].

The bulk tachyon of the closed bosonic string is the oldest known closed string tachyon. It remains the most mysterious one and there is no convincing analysis of the associated instability. The analogy with open strings, however, suggests a fairly dramatic possibility. In open bosonic string in the background of a spacefilling D-brane, the tachyon potential has a critical point that represents spacetime without the D-brane and thus without physical open string excitations. In an analogous closed string tachyon vacuum one would expect no closed string excitations. Without gravity excitations spacetime ceases to be dynamical and it would seem that, for all intents and purposes, it has dissappeared.

There has been no consensus that such a closed string tachyon vacuum exists. In fact, no analysis of the closed string tachyon potential (either in the CFT approach or in the SFT approach) has provided concrete evidence of a vacuum with non-dynamical spacetime. Since the analogous open string tachyon vacuum shows up quite clearly in the open string field theory computation of the potential it is natural to consider the corresponding calculation in closed string field theory (CSFT) [7, 8].

The quadratic and cubic terms in the closed string tachyon potential are well known [9, 10]:

$$\kappa^2 \mathbb{V}_0^{(3)} = -t^2 + \frac{6561}{4096}\, t^3 \,, \qquad (\alpha' = 2)\,. \tag{1.1}$$

These terms define a critical point analogous to the one that turns out to represent the tachyon vacuum in the open string field theory. In open string field theory higher level computations make the vacuum about 46% deeper. Since CSFT is nonpolynomial, it is natural to investigate the effect of the quartic term in the potential. This term was found to be [11, 12]

$$\kappa^2 V_0^{(4)} = -3.0172\, t^4 \,. \tag{1.2}$$

This term is so large and negative that $\mathbb{V}_0^{(3)} + V_0^{(4)}$ has no critical point. In fact, the quartic term in the *effective* tachyon potential (obtained by integrating out massive fields) is even a bit larger [11]. The hopes of identifying a reliable critical point in the closed string tachyon potential were dashed[1].

Recent developments inform our present analysis. The tachyon potential must include all fields that are sourced by the zero-momentum tachyon. As discussed in [14], this includes massless closed string states that are built from ghost oscillators, in particular, the zero-momentum ghost-dilaton state $(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})|0\rangle$. The search for a critical point cannot be carried out consistently without including the ghost dilaton. Computations of quartic vertices coupling dilatons, tachyons, and other massive fields are now possible due to the work of Moeller [12] and have been done to test the marginality of matter and dilaton operators [15, 16].

As we explain now, ghost-dilaton couplings to the tachyon restore the critical point in the potential. The key effect can be understood from the cubic and quartic couplings

$$\kappa^2 V(t,d) = -\frac{27}{32} t\, d^2 + 3.8721\, t^3 d + \cdots \,. \tag{1.3}$$

The cubic coupling plays no role as long as we only consider cubic interactions: d can be set consistently to zero. The quartic coupling is linear in d. Once included, the equation of motion for the dilaton can only be satisfied if the dilaton acquires an expectation value. Solving for the dilaton one finds $d = 2.2944\, t^2$ and substituting back,

$$\kappa^2 V(t,d) = 4.4422\, t^5 + \cdots \,. \tag{1.4}$$

[1] In the effective open string tachyon potential a negative quartic term also destroys the cubic critical point. Nevertheless, the critical point can be gleaned using Pade-approximants [13]. For closed strings, however, the quartic term is too large: for a potential $v(t) = v_2 t^2 + v_3 t^3 + v_4 t^4$, with $v_2, v_4 < 0$, the approximant formed by the ratio of a cubic and a linear polynomial fails to give a critical point when $v_2 v_4 \geq v_3^2$.

This positive quintic term suffices to compensate the effects of (1.2) and restores the critical point. Our computations include additional couplings and the effect of massive fields as well. The critical point persists and may be reliable, although more work is needed to establish this convincingly.

In order to interpret the critical point we raise and answer a pair of questions. The ghost-dilaton has a positive expectation value at the critical point. Does this correspond to stronger or weaker string coupling ? We do a detailed comparison of quadratic and cubic terms in the closed string field theory action and in the low-energy effective field theory action. The conclusion is that the positive dilaton expectation value corresponds to *stronger* coupling. In our solution the ghost-dilaton is excited but the scalar operator $c\bar{c}\,\partial X \cdot \bar{\partial} X$, sometimes included in the dilaton vertex operator, is not. We ask: Is the string metric excited? Is the Einstein metric excited? These questions are only well-defined at the linearized level, but the answers are clear: the string metric does *not* change, but the Einstein metric does. We take the opportunity to explain the relations between the four kinds of "dilatons" that are used in the literature: the ghost-dilaton, the matter-dilaton, the dilaton, and the dilaton of the older literature. It is noted that one cannot define unambiguously a dilaton vertex operator unless one specifies which metric is left invariant; conversely, the metric vertex operator is only determined once one specifies which dilaton is left invariant.

In a companion paper [17] we attempted to gain insight into the tachyon vacuum by considering the rolling solutions[2] of a low-energy effective action for the string metric $g_{\mu\nu}$, the tachyon T, and the dilaton Φ:

$$S_\sigma = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\, e^{-2\Phi}\Big(R + 4(\partial_\mu \Phi)^2 - (\partial_\mu T)^2 - 2V(T)\Big)\,. \tag{1.5}$$

This action, suggested by the beta functions of sigma models with background fields [22], is expected to capture at least some of the features of string theory solutions. The potential is tachyonic: $V(T) = -\frac{1}{2}m^2T^2 + \mathcal{O}(T^3)$, but is otherwise left undetermined. We found that solutions in which the tachyon begins the rolling process always have *constant* string metric for all times — consistent with the type of the SFT critical point. The dilaton, moreover, grows in time throughout the evolution — consistent with the larger dilaton vev in the SFT critical point. Rather generally, the solution becomes singular in finite time: the dilaton runs to infinity and the string coupling becomes infinite. Alternatively, the Einstein metric crunches up and familiar spacetime no longer exists. This seems roughly consistent with the idea that the tachyon vacuum does not have a fluctuating spacetime.

Perhaps the most subtle point concerns the value of the on-shell action. In the open string field theory computation of the tachyon potential, the value of the action (per unit spacetime volume) is energy density. The tachyon conjectures are in fact formulated in terms of energy densities at the perturbative and the non-perturbative vacuum [1]. Since the tree-level cosmological constant in closed string theory is zero, the value of the action at

[2]Rolling solutions have long been considered using Liouville field theory to provide conformal invariant sigma model with spacetime background fields that typically include a linear dilaton and a constant string metric [18 – 21].

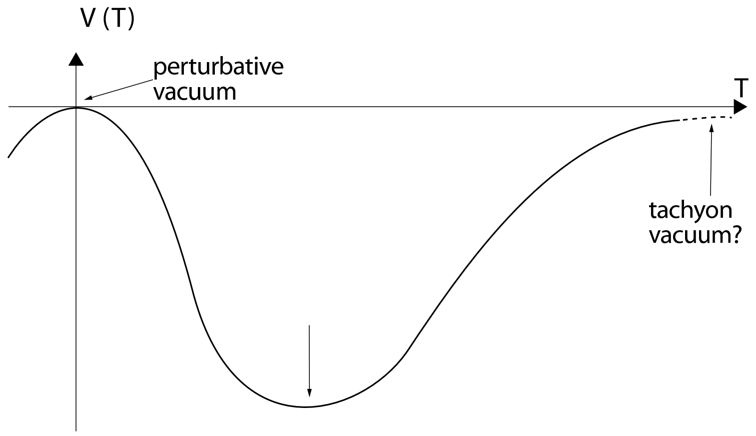

Figure 1: A sketch of a closed string tachyon potential consistent with present evidence. The perturbative vacuum is at $T = 0$. The closed string tachyon vacuum would be the critical point with zero cosmological term, shown here at $T \to \infty$ (in CSFT this point corresponds to finite tachyon vev). A critical point with negative cosmological constant cannot provide a spacetime independent tachyon vacuum.

the perturbative closed string vacuum is zero. We ask: What is the value of the potential, or action (per unit volume) at the critical point ? The low-energy action (1.5) suggests a surprising answer. Consider the associated equations of motion:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - (\partial_\mu T)(\partial_\nu T) &= 0\,, \\ \nabla^2 T - 2(\partial_\mu \Phi)(\partial^\mu T) - V'(T) &= 0\,, \\ \nabla^2 \Phi - 2(\partial_\mu \Phi)^2 - V(T) &= 0\,. \end{aligned} \tag{1.6}$$

If the fields acquire *constant* expectation values we can satisfy the tachyon equation if the expectation value T_* is a critical point of the potential: $V'(T_*) = 0$. The dilaton equation imposes an additional constraint: $V(T_*) = 0$, the potential must itself vanish. This is a reliable constraint that follows from a simple fact: in the action the dilaton appears without derivatives only as a multiplicative factor. This fact remains true after addition of α' corrections of all orders. It may be that $V(T)$ has a critical point T_0 with $V(T_0) < 0$, but this cannot be the tachyon vacuum. The effective field equations imply that a vacuum with spacetime independent expectation values has *zero* action.

The action (1.5) can be evaluated on-shell using the equations of motion. One finds

$$S_{\text{on-shell}} = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g}\, e^{-2\Phi} \big(-4V(T)\big)\,. \tag{1.7}$$

In rolling solutions the action density changes in time but, as $\Phi \to \infty$ at late times the action density goes to zero [17]. This also suggests that the tachyon vacuum is a critical point with zero action.

In figure 1 we present the likely features of the tachyon potential. The unstable perturbative vacuum $T = 0$ has zero cosmological constant, and so does the tachyon vacuum $T = \infty$. The infinite value of T is suggested by the analogous result in the effective open string theory tachyon potential (see conclusions). In SFT the tachyon vacuum appears for finite values of the fields, but the qualitative features would persist. The potential is qualitatively in the class used in cyclic universe models [23].

In our calculations we find some evidence that the action density, which is negative, may go to zero as we increase the accuracy of the calculation. To begin with, the value Λ_0 of the action density at the critical point of the cubic tachyon potential (1.1) may be argued to be rather small. It is a cosmological term about seventy times smaller than the "canonical" one associated with $D = 2$ non-critical string theory (see [3], footnote 5). Alternatively, Λ_0 is only about 4% of the value that would be obtained using the on-shell coupling of three tachyons to calculate the cubic term. The inclusion of cubic interactions of massive fields makes the action density about 10% more negative. This shift, smaller than the corresponding one in open string field theory, is reversed once we include the dilaton quartic terms. In the most accurate computation we have done, the action density is down to 60% of Λ_0. Additional computations are clearly in order.

As a by-product of our work, we investigate large dilaton deformations in CSFT. For ordinary marginal deformations the description reaches an obstruction for some finite critical value of the string field marginal parameter [24, 25]. The critical value is stable under level expansion, and the potential for the marginal field (which should vanish for infinite level) is small. For the dilaton, however, the lowest-order obstruction is not present [16]. We carry this analysis to higher order and no reliable obstructions are found: critical values of the dilaton jump wildly with level and appear where the dilaton potential is large and cannot be trusted. This result strengthens the evidence that CSFT can describe backgrounds with arbitrarily large variations in the string coupling. If the infinite string coupling limit is also contained in the configuration space it may be possible to define M-theory using type-IIA superstring field theory.

Let us briefly describe the contents of this paper. In section 2 we reconsider the universality arguments [14] that require the inclusion of the ghost-dilaton, exhibit a world-sheet parity symmetry that allows a sizable truncation of the universal space, and note that universality may apply in circumstances significantly more general that originally envisioned [26]. Our computational strategy for the tachyon potential, motivated by the results of [15, 16], goes as follows. We compute *all* quadratic and cubic terms in the potential including fields up to level four. We then begin the inclusion of quartic terms and obtain complete results up to quartic interactions of total level four. The results make it plausible that a critical point exists and that the value of the action density decreases in magnitude as the accuracy improves. In section 3 we find the linearized relations between the metric, dilaton, and tachyon closed string fields and the corresponding fields in the sigma-model approach to string theory. These relations allow us to establish that the dilaton vev at the critical point represents an increased string coupling and that the string field at the critical point does not have a component along the vertex operator for the string metric. We discuss the vertex operators associated with the various definitions of the dilaton, determine the nonlinear field relations between the string field theory and effective field theory dilatons and tachyons to quadratic order and at zero-momentum, and examine large dilaton deformations. In the concluding section we discuss additional considerations that suggest the existence of the tachyon vacuum. These come from non-critical string theory, p-adic strings, and sigma model arguments. Finally, the details of the nontrivial computations of quartic couplings are given in the appendix.

2. Computation of the tachyon potential

In this section we present the main computations of this paper. We begin by introducing the string field relevant for the calculation of the tachyon potential, giving a detailed discussion of universality. This string field contains the tachyon, at level zero, the ghost-dilaton, at level two, and massive fields at higher even levels. We then give the quadratic and cubic couplings for the string field restricted to level four and calculate the critical point. Finally, we give the quartic couplings at level zero, two, and four. The critical point survives the inclusion of quartic interactions and becomes more shallow — consistent with the conjecture that the tachyon vacuum has zero action.

The computations use the closed string field action [7, 8, 3], which takes the form

$$S = -\frac{2}{\alpha'}\Big(\frac{1}{2}\langle\Psi|c_0^- \, Q|\Psi\rangle + \frac{\kappa}{3!}\{\Psi, \Psi\, , \Psi\} + \frac{\kappa^2}{4!}\{\Psi,\, \Psi, \Psi, \Psi\} + \cdots\Big)\,. \tag{2.1}$$

The string field Ψ lives on $\mathcal{H}$, the ghost number two state space of the full CFT restricted to the subspace of states that satisfy

$$(L_0 - \bar{L}_0)|\Psi\rangle = 0 \qquad \text{and} \qquad (b_0 - \bar{b}_0)|\Psi\rangle = 0\,. \tag{2.2}$$

The BRST operator is $Q = c_0 L_0 + \bar{c}_0 \bar{L}_0 + \ldots$, where the dots denote terms independent of c_0 and of $\bar{c}_0$. Moreover, $c_0^{\pm} = \frac{1}{2}(c_0 \pm \bar{c}_0)$, and we normalize correlators using $\langle 0|c_{-1}\bar{c}_{-1}c_0^- c_0^+ c_1\bar{c}_1|0\rangle = 1$. All spacetime coordinates are imagined compactified with the volume of spacetime set equal to one.

2.1 Tachyon potential universality and the ghost-dilaton

The universality of the closed string tachyon potential was briefly discussed in [14], where it was also noted that the ghost number two universal string field that contains the tachyon should include the zero-momentum ghost-dilaton state $(c_1c_{-1}-\bar{c}_1\bar{c}_{-1})|0\rangle$. In here we review the universality argument and extend it slightly, offering the following observations:

- The ghost-dilaton must be included because closed string field theory is not cubic.

- A world-sheet parity symmetry of closed string field theory can be used to restrict the universal subspace.

- The arguments of [14] do not apply directly to general CFT's, linear dilaton backgrounds, for example. If the closed string background is defined by a general matter CFT, solutions on the universal subspace may still be solutions, but there is no tachyon potential [26].

The original idea in universality is to produce a subdivision of all the component fields of the string field theory into two disjoint sets, a set $\{t_i\}$ that contains the zero-momentum tachyon and a set $\{u_a\}$ such that the string field action $S(t_i, u_a)$ contains no term with a single u-type field. It is then consistent to search for a solution of the equations of motion that assumes $u_a = 0$ for all a.

To produce the desired set $\{t_i\}$ we assume that the matter CFT is such that X^0 is the usual negative-metric field with associated conserved momentum k_0 and the rest of the matter CFT is unitary. The state space $\mathcal{H}$ (see (2.2)) is then divided into three disjoint vector subspaces $\mathcal{H}_1, \mathcal{H}_2$, and $\mathcal{H}_3$. One has $\mathcal{H}_i = \mathcal{M}_i \otimes |\mathcal{G}\rangle$, where $|\mathcal{G}\rangle$ denotes a state built with ghost and antighost oscillators only and $\mathcal{M}_1, \mathcal{M}_2$, and $\mathcal{M}_3$ are disjoint subspaces of the matter CFT whose union gives the total matter CFT state space:

$$
\begin{aligned}
&\mathcal{M}_1: \ \text{the } \mathrm{SL}(2,\mathbb{C}) \text{ vacuum } |0\rangle \text{ and descendents},\\
&\mathcal{M}_2: \ \text{states with } k_0 \neq 0\,,\\
&\mathcal{M}_3: \ \text{primaries with } k_0 = 0 \text{ but different from } |0\rangle \text{ and descendents}.
\end{aligned} \tag{2.3}
$$

In the above, primary and descendent refers to the matter Virasoro operators. Note that the primaries in $\mathcal{M}_3$ have positive conformal dimension. The BRST operator preserves the conditions (2.2), and since it is composed of ghost oscillators and matter Virasoro operators, it maps each $\mathcal{H}_i$ into itself. Finally, the spaces $\mathcal{H}_i$ are orthogonal under the BPZ inner product; they only couple to themselves.

The claim is that the set $\{t_i\}$ is in fact $\mathcal{H}_1$, the states built upon the zero momentum vacuum. The "tachyon potential" is the string action evaluated for $\mathcal{H}_1$.

We first note that because of momentum conservation fields in $\mathcal{H}_2$ cannot couple linearly to fields in $\mathcal{H}_1$. The fields in $\mathcal{H}_3$ cannot couple linearly to the fields in $\mathcal{H}_1$ either. They cannot do so through the kinetic term because the BRST operator preserves the space and $\mathcal{H}_1$ and $\mathcal{H}_3$ are BPZ orthogonal. We also note that the matter correlator in the n-string vertex does not couple $n-1$ vacua $|0\rangle$ from $\mathcal{H}_1$ to a matter primary from $\mathcal{H}_3$: this is just the one-point function of the primary in $\mathcal{H}_3$, which vanishes because the state has non-zero dimension. The (matter) Virasoro conservation laws on the vertex then imply that the coupling of any $(n-1)$ states in $\mathcal{H}_1$ to a state in $\mathcal{H}_3$ must vanish. This completes the proof that $\mathcal{H}_1$ is the subspace for tachyon condensation.

The space $\mathcal{H}_1$ can be written as

$$
\mathrm{Span}\Big\{ L^m_{-j_1}\ldots L^m_{-j_p}\,\bar{L}^m_{-\bar{j}_1}\ldots \bar{L}^m_{-\bar{j}_{\bar{p}}} b_{-k_1}\ldots b_{-k_q}\,\bar{b}_{-\bar{k}_1}\ldots \bar{b}_{-\bar{k}_{\bar{q}}}\,c_{-l_1}\ldots c_{-l_r}\;\bar{c}_{-\bar{l}_1}\ldots \bar{c}_{-\bar{l}_{\bar{r}}}\,|0\rangle \Big\}\,, \tag{2.4}
$$

where

$$
j_1 \geq j_2 \geq \cdots \geq j_p\,, \quad j_i \geq 2\,, \qquad \bar{j}_1 \geq \bar{j}_2 \geq \cdots \geq \bar{j}_{\bar{p}}\,, \quad \bar{j}_i \geq 2\,, \tag{2.5}
$$

as well as

$$
k_i, \bar{k}_i \geq 2\,, \qquad l_i, \bar{l}_i \geq -1\,, \qquad \text{and} \qquad r + \bar{r} - q - \bar{q} = 2\,. \tag{2.6}
$$

Finally, the states above must also be annihilated by $L_0 - \bar{L}_0$ as well as $b_0 - \bar{b}_0$.

There is a reality condition on the string field [7]: its BPZ and hermitian conjugates must differ by a sign. We show now that this condition is satisfied by all the states in (2.4), so the coefficients by which they are multiplied in the universal string field (the zero-momentum spacetime fields) must be real. Suppose a state is built with p ghost oscillators and $p-2$ antighost oscillators. The BPZ and hermitian conjugates differ by the product of two factors: a $(-1)^p$ from the BPZ conjugation of the ghost oscillators and a $(-1)^{(2p-2)(2p-1)/2} = (-1)^{p-1}$ from the reordering of oscillators in the hermitian conjugate. The product of these two factors is minus one, as we wanted to show.

In open string theory twist symmetry, which arises from world-sheet parity, can be used to further restrict the universal subspace constructed from matter Virasoro and ghost oscillators. In the case of closed string theory the world-sheet parity transformation that exchanges holomorphic and antiholomorphic sectors is the relevant symmetry.[3] World-sheet parity is not necessarily a symmetry of arbitrary matter CFT's, but it is a symmetry in the universal subspace: correlators are complex conjugated when we exchange holomorphic and antiholomorphic Virasoro operators as $T(z) \leftrightarrow \bar{T}(\bar{z})$. More precisely, we introduce a $\star$-conjugation, a map of $\mathcal{H}_1$ to $\mathcal{H}_1$ that is an involution. In a basis of Virasoro modes $\star$ can be written explicitly as the map of states

$$\star : \quad A\, L_{-i_1} \cdots L_{-i_n}\, \bar{L}_{-j_1} \cdots \bar{L}_{-j_n} |0\rangle \quad \rightarrow \quad A^*\, \bar{L}_{-i_1} \cdots \bar{L}_{-i_n}\, L_{-j_1} \cdots L_{-j_n} |0\rangle\,, \qquad (2.7)$$

where A is a constant and A^* denotes its complex conjugate. Given the operator/state correspondence, the above defines completely the star operation $\star :\ \mathcal{O} \rightarrow \mathcal{O}^\star$ on vertex operators for vacuum descendents. It results in the following property for the correlator of n such operators placed at n points on a Riemann surface:

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = \langle \mathcal{O}_1^\star \ldots \mathcal{O}_n^\star \rangle^*\,. \qquad (2.8)$$

In the ghost sector of the CFT a small complication with signs arises because the basic correlator is odd under the exchange of holomorphic and anti-holomorphic sectors:

$$\langle\, c(z_1)c(z_2)c(z_3)\, \bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3) \rangle = -\langle\, \bar{c}(\bar{z}_1)\bar{c}(\bar{z}_2)\bar{c}(\bar{z}_3)\, c(w_1)c(w_2)c(w_3)\, \rangle^*\,. \qquad (2.9)$$

Since two-point functions of the ghost fields are complex conjugated by the exchanges $c(z) \leftrightarrow \bar{c}(\bar{z})$ and $b(z) \leftrightarrow \bar{b}(\bar{z})$, it follows from (2.9) that performing these exchanges on an *arbitrary* correlator of ghost and antighost fields will give minus the complex conjugate of the original correlator. We will define $\star$-conjugation in the ghost sector by:

$$\star : \quad A\, c_{i_1} \cdot\cdot c_{i_n}\, b_{j_1} \cdot\cdot b_{j_m}\, \bar{c}_{k_1} \cdot\cdot \bar{c}_{k_r}\, \bar{b}_{l_1} \cdot\cdot \bar{b}_{l_s} |0\rangle \quad \rightarrow \quad A^*\, \bar{c}_{i_1} \cdot\cdot \bar{c}_{i_n}\, \bar{b}_{j_1} \cdot\cdot \bar{b}_{j_m}\, c_{k_1} \cdot\cdot c_{k_r}\, b_{l_1} \cdot\cdot b_{l_s} |0\rangle\,. \quad (2.10)$$

For a general state Ψ of the universal subspace we define $\Psi^\star$ to be the state obtained by the simultaneous application of (2.7) and (2.10). It is clear from the above discussion that the correlators satisfy

$$\langle \Psi_1\, \Psi_2 \ldots \Psi_n \rangle = -\langle \Psi_1^\star\, \Psi_2^\star \ldots \Psi_n^\star \rangle^*\,, \qquad \Psi_i \in \mathcal{H}_1\,. \qquad (2.11)$$

[3]We thank A. Sen for discussions that led us to construct the arguments presented below.

We now define the action of the world-sheet parity operation $\mathcal{P}$ on arbitrary states of the universal subspace:

$$\mathcal{P}\Psi \equiv -\Psi^\star\,, \qquad \Psi \in \mathcal{H}_1\,. \tag{2.12}$$

We claim that the string field theory action, restricted to $\mathcal{H}_1$, is $\mathcal{P}$ invariant:

$$S(\Psi) = S(\mathcal{P}\Psi)\,, \qquad \text{for } \Psi \in \mathcal{H}_1\,. \tag{2.13}$$

First consider the invariance of the cubic term. Using (2.12) and (2.11) we have

$$\langle \mathcal{P}\Psi\,, \mathcal{P}\Psi\,, \mathcal{P}\Psi\rangle = -\langle \Psi^\star\,, \Psi^\star\,, \Psi^\star\rangle = \langle \Psi\,, \Psi\,, \Psi\rangle^* = \langle \Psi\,, \Psi\,, \Psi\rangle\,, \tag{2.14}$$

where in the last step we used the reality of the string field action. The kinetic term of the action is also invariant. First note that $(c_0^- Q\Psi)^\star = -c_0^- Q\Psi^\star$. It then follows that

$$\langle \mathcal{P}\Psi\,, c_0^- Q\mathcal{P}\Psi\rangle = \langle \Psi^\star\,, c_0^- Q\Psi^\star\rangle = -\langle \Psi^\star\,, (c_0^- Q\Psi)^\star\rangle = \langle \Psi\,, c_0^- Q\Psi\rangle^* = \langle \Psi\,, c_0^- Q\Psi\rangle\,. \tag{2.15}$$

For higher point interactions, the invariance follows because the antighost insertions have the appropriate structure. Each time we add a new string field we must add two antighost insertions. For the case of quartic interactions they take the form of two factors $\mathcal{B}\mathcal{B}^\star$ (see eq. (A.3)). Since $(\mathcal{B}\mathcal{B}^\star)^\star = -\mathcal{B}\mathcal{B}^\star$, the extra minus sign cancels against the minus sign from the extra string field. This can be seen to generalize to higher order interactions using the forms of the off-shell amplitudes discussed in section 6 of [10]. This completes our proof of (2.13).

Since $\mathcal{P}^2 = 1$ the space $\mathcal{H}_1$ can be divided into two disjoint subspaces: the space $\mathcal{H}_1^+$ of states with $\mathcal{P} = 1$ and the space $\mathcal{H}_1^-$ of states with $\mathcal{P} = -1$:

$$\begin{aligned} \mathcal{P}(\Psi_+) &= +\Psi_+, \qquad \Psi_+ \in \mathcal{H}_1^+\,, \\ \mathcal{P}(\Psi_-) &= -\Psi_-, \qquad \Psi_- \in \mathcal{H}_1^-\,. \end{aligned} \tag{2.16}$$

It follows from the invariance of the action that no term in the action can contain just one state in $\mathcal{H}_1^-$. We can therefore restrict ourselves to the subspace $\mathcal{H}_1^+$ with positive parity.

The string field is further restricted by using a gauge fixing condition. The computation of the potential is done in the Siegel gauge, which requires states to be annihilated by $b_0+\bar{b}_0$. To restrict ourselves to the Siegel gauge we take the states in (2.4) that have neither a c_0 nor a $\bar{c}_0$.

The Siegel gauge fixes the gauge symmetry completely for the massive levels, but does not quite do the job at the massless level. There are two states with $L_0 = \bar{L}_0 = 0$ in $\mathcal{H}_1$ that are in the Siegel gauge:

$$(c_1 c_{-1} - \bar{c}_1\bar{c}_{-1})|0\rangle \qquad \text{and} \qquad (c_1 c_{-1} + \bar{c}_1\bar{c}_{-1})|0\rangle\,. \tag{2.17}$$

The first state is the ghost dilaton and it is proportional to $Q(c_0-\bar{c}_0)|0\rangle$. Since $(c_0-\bar{c}_0)|0\rangle$ is not annihilated by $b_0-\bar{b}_0$ the gauge parameter is illegal and the ghost dilaton is not trivial. The second state is proportional to $Q(c_0+\bar{c}_0)|0\rangle$, so it is thus trivial at the linearized level. Although trivial at the linearized level, one may wonder if the triviality holds for large fields. Happily, we need not worry: the state is $\mathcal{P}$ odd, so it need not be included in the calculation. The ghost-dilaton, because of the relative minus sign between the two terms, is $\mathcal{P}$ even and it is included.

Had the closed string field theory been cubic we could have discarded the ghost-dilaton state and all other states with asymmetric left and right ghost numbers. We could restrict $\mathcal{H}_1^+$ to fields of ghost number $(G,\bar{G})=(1,1)$. Indeed, the cubic vertex cannot couple two $(1,1)$ fields to anything except another $(1,1)$ field. Moreover, in the Siegel gauge $c_0^- Q$ acts as an operator of ghost number $(1,1)$, so again, no field with asymmetric ghost numbers can couple linearly. The quartic and higher order interactions in CSFT have antighost insertions that do not have equal left and right ghost numbers. It follows that these higher order vertices can couple the ghost-dilaton to $(1,1)$ fields. Indeed, the coupling of a dilaton to three tachyons does not vanish. We *cannot* remove from $\mathcal{H}_1^+$ the dilaton, nor other states with asymmetric left and right ghost numbers.

The construction of the universal string field and action presented here does not work fully if the matter CFT contains a linear dilaton background. Momentum conservation along the corresponding coordinate is anomalous and one cannot build an action with states of zero momentum only: the action restricted to $\mathcal{H}_1$ is identically zero. There would be no universal "potential" in $\mathcal{H}_1$. It appears rather likely, however, that any *solution* in the universal subspace would still be a solution in a linear dilaton background. In fact, any solution in the universal subspace may be a solution for string field theory formulated with a general matter CFT [26].

We conclude this section by writing out the string field for the first few levels. The level ℓ of a state is defined by $\ell = L_0+\bar{L}_0+2$. The level zero part of the string field is

$$|\Psi_0\rangle = t\, c_1\bar{c}_1|0\rangle\,. \tag{2.18}$$

Here t is the zero-momentum tachyon. The level two part of the string field is

$$|\Psi_2\rangle = d\,(c_1c_{-1}-\bar{c}_1\bar{c}_{-1})|0\rangle\,. \tag{2.19}$$

Here d is the zero momentum ghost-dilaton. It multiplies the only state of $\mathcal{P}=+1$ at this level. At level four there are four component fields:

$$\begin{aligned}|\Psi_4\rangle = \Big(&f_1\, c_{-1}\bar{c}_{-1} + f_2\, L_{-2}c_1\,\bar{L}_{-2}\bar{c}_1 + f_3\,(L_{-2}c_1\bar{c}_{-1}+c_{-1}\,\bar{L}_{-2}\bar{c}_1) + \\ &+ g_1\,(b_{-2}c_1\,\bar{c}_{-2}\bar{c}_1 - c_{-2}c_1\,\bar{b}_{-2}\bar{c}_1)\Big)|0\rangle\,.\end{aligned} \tag{2.20}$$

Note that the states coupling to the component fields all have $\mathcal{P}=+1$ and that g_1 couples to a state with asymmetric left and right ghost numbers. In this paper we will not use higher level terms in the string field.

With $\alpha'=2$ the closed string field potential V associated with the action in (2.1) is

$$\kappa^2 V = \frac{1}{2}\langle\Psi|c_0^-\,Q|\Psi\rangle + \frac{1}{3!}\{\Psi,\Psi,\Psi\} + \frac{1}{4!}\{\Psi,\Psi,\Psi,\Psi\}+\cdots\,. \tag{2.21}$$

Here $|\Psi\rangle = |\Psi_0\rangle + |\Psi_2\rangle + |\Psi_4\rangle + \cdots$. Our computations will not include quintic and higher order interactions in the string action.

2.2 The quadratic and cubic terms in the potential

Let us now consider the potential including only the kinetic and cubic terms in (2.21). To level zero:

$$\kappa^2 V_0^{(2)} = -t^2\,, \qquad \kappa^2 V_0^{(3)} = \frac{6561}{4096}\, t^3\,. \tag{2.22}$$

All potentials introduced in this subsection have a superscript that gives the order of the interaction (two for quadratic, three for cubic, and so on), and a subscript that gives the level (defined by the sum of levels of fields in the interaction). The next terms arise at level four, where we have couplings of the tachyon to the square of the dilaton and couplings of the level four fields to the tachyon squared:

$$\kappa^2\, V_4^{(3)} = -\frac{27}{32}\, d^2\, t + \left(\frac{3267}{4096}\, f_1 + \frac{114075}{4096}\, f_2 - \frac{19305}{2048} f_3\right) t^2\,. \tag{2.23}$$

At level six we can couple a level four field, a dilaton, and a tachyon. Only level four fields with $G \neq \overline{G}$ can have such coupling, so we find:

$$\kappa^2\, V_6^{(3)} = -\frac{25}{8}\, g_1\, t\, d\,. \tag{2.24}$$

At level eight there are two kinds of terms. First, we have the kinetic terms for the level four fields:

$$\kappa^2\, V_8^{(2)} = {f_1}^2 + 169\, {f_2}^2 - 26\, {f_3}^2 - 2\, {g_1}^2\,. \tag{2.25}$$

Second, we have the cubic interactions:

$$\begin{aligned}\kappa^2\, V_8^{(3)} \;=\; & -\frac{1}{96}\, f_1\, d^2 - \frac{4225}{864}\, f_2\, d^2 + \frac{65}{144}\, f_3\, d^2 + \frac{361}{12288}\, {f_1}^2\, t + \frac{511225}{55296}\, f_1\, f_2\, t + \\ & + \frac{57047809}{110592}\, {f_2}^2\, t + \frac{511225}{55296}\, {f_3}^2\, t - \frac{49}{24}\, {g_1}^2\, t - \frac{13585}{9216}\, f_1\, f_3\, t - \\ & - \frac{5400395}{27648}\, f_2\, f_3\, t + \frac{143507}{18432}\, {f_3}^2\, t\,.\end{aligned} \tag{2.26}$$

As we can see, these are of two types: couplings of a level four field to two dilatons (first line) and couplings of two level four fields to a tachyon (second and third lines).

The terms at level 10 couple two level four fields and a dilaton. Because of ghost number conservation, one of the level four fields must have $G \neq \overline{G}$:

$$\kappa^2\, V_{10}^{(3)} = -\frac{25}{5832}\big(361\, f_1 + 4225\, f_2 - 2470\, f_3\big)\, d\, g_1\,. \tag{2.27}$$

Finally, at level 12 we have the cubic couplings of three level-four fields:

$$\begin{aligned}\kappa^2\, V_{12}^{(3)} \;=\; & \frac{1}{4096}\, f_1^3 + \frac{1525225}{8957952}\, f_1^2 f_2 - \frac{1235}{55296} f_1^2\, f_3 + \frac{6902784889}{80621568} f_1 f_2^2 - \\ & - \frac{102607505}{6718464} f_1 f_2 f_3 + \frac{1884233}{2239488} f_1 f_3^2 + \frac{74181603769}{26873856} f_2^3 - \\ & - \frac{22628735129}{13436928}\, f_2^2 f_3 + \frac{4965049817}{20155392} f_2 f_3^2 - \frac{31167227}{3359232} f_3^3 - \\ & - \frac{961}{157464}\, f_1 g_1^2 - \frac{207025}{17496}\, f_2 g_1^2 + \frac{14105}{26244}\, f_3 g_1^2\,.\end{aligned} \tag{2.28}$$

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Potential	t	f_1	f_2	f_3	Action density
$\mathbb{V}_0^{(3)}$	0.41620	$--$	$--$	$--$	-0.05774
$\mathbb{V}_8^{(3)}$	0.43678	-0.06502	-0.00923	-0.02611	-0.06329
$\mathbb{V}_{12}^{(3)}$	0.43709	-0.06709	-0.00950	-0.02693	-0.06338

Table 1: Vacuum solution with cubic vertices only.

2.3 Tachyon vacuum with cubic vertices only

With cubic vertices only the dilaton expectation value is zero. In fact, only fields with $G = \overline{G} = 1$ can acquire nonvanishing expectation values. To examine the tachyon vacuum we define a series of potentials:

$$\begin{aligned}
\mathbb{V}_0^{(3)} &\equiv V_0^{(2)} + V_0^{(3)} \,, \\
\mathbb{V}_8^{(3)} &\equiv \mathbb{V}_0^{(3)} + V_4^{(3)} + V_6^{(3)} + V_8^{(2)} + V_8^{(3)} \,, \\
\mathbb{V}_{12}^{(3)} &\equiv \mathbb{V}_8^{(3)} + V_{10}^{(3)} + V_{12}^{(3)} \,.
\end{aligned} \tag{2.29}$$

A few observations are in order. In all of the above potentials we can set $d = g_1 = 0$. As a consequence, $V_6^{(3)}$ and $V_{10}^{(3)}$ do not contribute. Since the level-two dilaton plays no role, once we go beyond the tachyon we must include level four fields. The kinetic terms for these fields are of level eight, so $\mathbb{V}_8^{(3)}$ is the simplest potential beyond level zero. With level-four fields the next potential is $\mathbb{V}_{12}^{(3)}$.

The critical points obtained with the potentials $\mathbb{V}_0^{(3)}$, $\mathbb{V}_8^{(3)}$, and $\mathbb{V}_{12}^{(3)}$ are given in table 1. We call the value of the potential $\kappa^2\mathbb{V}$ at the critical point the action density. The values of the action density follow the pattern of open string theory. The original cubic critical point becomes deeper. It does so by about 10%, a value significantly smaller than the corresponding one in open string field theory.

2.4 Tachyon vacuum with cubic and quartic vertices

We can now examine the quartic terms in the potential. The associated potentials are denoted with a superscript (4) for quartic and a subscript that gives the sum of levels of the fields that enter the term. The quartic self-coupling of tachyons has been calculated in [11, 12]:

$$\kappa^2 V_0^{(4)} = -3.0172\, t^4 \,. \tag{2.30}$$

With total level two we have a coupling of three tachyons and one dilaton. This is calculated in appendix A.2 and the result is

$$\kappa^2 V_2^{(4)} = 3.8721\, t^3 d \,. \tag{2.31}$$

With total level four there is the coupling of two tachyons to two dilatons (appendix A.2) and the coupling of three tachyons to any of the level-four fields (appendix A.3):

$$\kappa^2 V_4^{(4)} = 1.3682\, t^2 d^2 + t^3\big(-0.4377\, f_1 - 56.262\, f_2 + 13.024\, f_3 + 0.2725\, g_1\big) \,. \tag{2.32}$$

Potential	t	d	f_1	f_2	f_3	g_1	Action density
$\mathbb{V}_{12}^{(3)}$	0.43709	0	-0.06709	-0.00950	-0.02693	$--$	-0.06338
$\mathbb{V}_{0}^{(4)}$	$--$	$--$	$--$	$--$	$--$	$--$	$--$
$\mathbb{V}_{2}^{(4)}$	0.33783	0.49243	-0.08007	-0.00619	-0.02607	-0.10258	-0.05806
$\mathbb{V}_{4}^{(4)}$	0.24225	0.45960	-0.04528	-0.00140	-0.01233	-0.07249	-0.03382

Table 2: Vacuum solution with cubic and quartic vertices. We see that the magnitude of the action density becomes smaller as we begin to include the effects of quartic couplings.

With total level six there are three types of interactions: a tachyon coupled to three dilatons, two tachyons coupled to a dilaton and a level-four field, and three tachyons coupled to a level-six field. We have only computed the first one (appendix A.2):

$$\kappa^2 V_6^{(4)} = -0.9528\, td^3 + \cdots . \tag{2.33}$$

The terms that have not been computed are indicated by the dots. Finally, the quartic self-coupling of dilatons was computed in [16], where it played a central role in the demonstration that the effective dilaton potential has no quartic term:

$$\kappa^2 V_8^{(4)} = -0.1056\, d^4 + \cdots . \tag{2.34}$$

We use the dots to indicate the additional level eight interactions that should be computed.

Let us now consider the potentials that can be assembled using the above contributions. We use the following strategy: we include cubic vertices to the highest possible level and then begin to introduce the quartic couplings level by level. The most accurate potential with quadratic and cubic terms that we have is $\mathbb{V}_{12}^{(3)}$ and the tachyon vacuum it contains appears in the last line of table 1. The lowest order quartic potential that we use is therefore:

$$\mathbb{V}_0^{(4)} \equiv \ \mathbb{V}_{12}^{(3)} + V_0^{(4)} . \tag{2.35}$$

This potential has a familiar difficulty: the quartic self-coupling of the tachyon is so strong that the critical point in the potential disappears. As we have argued, once additional terms are included the critical point in the potential reappears. The higher level potentials are defined by including progressively higher level quartic interactions:

$$\begin{aligned} \mathbb{V}_2^{(4)} &\equiv \mathbb{V}_0^{(4)} + V_2^{(4)} , \\ \mathbb{V}_4^{(4)} &\equiv \mathbb{V}_2^{(4)} + V_4^{(4)} . \end{aligned} \tag{2.36}$$

Since our computations of $V_6^{(4)}$ and $V_8^{(4)}$ are incomplete, the results that follow from $\mathbb{V}_6^{(4)} \equiv \mathbb{V}_4^{(4)} + V_6^{(4)}$ and $\mathbb{V}_8^{(4)} \equiv \ \mathbb{V}_6^{(4)} + V_8^{(4)}$ cannot be trusted.

We are now in a position to calculate the critical points of the potentials $\mathbb{V}^{(4)}$. In our numerical work we input the cubic coefficients as fractions and the quartic coefficients as the exact decimals given above (so the t^4 coefficient is treated as exactly equal to 3.0172.) Our results are given in table 2. For ease of comparison, we have included the cubic results for $\mathbb{V}_{12}^{(3)}$ as the first line. Furthermore, we include a line for $\mathbb{V}_0^{(4)}$ even though there is no critical point. The next potential is $\mathbb{V}_2^{(4)}$ which contains only the additional coupling t^3d. The significant result is that the critical point reappears and can be considered to be a (moderate) deformation of the critical point obtained with $\mathbb{V}_{12}^{(3)}$. Indeed, while there is a new expectation value for the dilaton (and for g_1), the expectation value of the tachyon does not change dramatically, nor do the expectation values for f_1, f_2, and f_3. The critical point becomes somewhat shallower, despite the destabilizing effects of the tachyon quartic self-couplings.

At the next level, where t^2d^2 and t^3M_4 (M_4 denotes a level-four field) terms appear, the critical point experiences some significant change. First of all, it becomes about 40% more shallow; the change is large and probably significant, given the expectation that the action density should eventually reach zero. The tachyon expectation changes considerably but the dilaton expectation value changes little. Due to the t^3M_4 terms the expectation values of some of the level four fields change dramatically.

Glancing at table 2, one notices that the tachyon expectation value is becoming smaller so one might worry that the critical point is approaching the perturbative vacuum. This is, of course, a possibility. If realized, it would imply that the critical point we have encountered is an artifact of level expansion. We think this is unlikely. Since the dilaton seems to be relatively stable, a trivial critical point would have to be a dilaton deformation of the perturbative vacuum, but such deformations have negative tachyon expectation values (see figure 2).

At this moment we do not have full results for higher levels. The computation of $\mathbb{V}_6^{(4)}$ would require the evaluation of couplings of the form t^2dM_4 and, in principle, couplings t^3M_6 of level-six fields, which we have not even introduced in this paper. The only additional couplings we know at present are td^3, which enters in $\mathbb{V}_6^{(4)}$ and d^4, which enters in $\mathbb{V}_8^{(4)}$ (see eqs. (2.33) and (2.34)). Despite lacking terms, we calculated the resulting vacua to test that no wild effects take place. The incomplete $\mathbb{V}_6^{(4)}$ leads to $t = 0.35426,\ d = 0.40763$ and an action density of -0.05553. The incomplete $\mathbb{V}_8^{(4)}$ leads to $t = 0.36853,\ d = 0.40222$ and an action density of -0.05836. In these results the action density has become more negative. Given the conjectured value of the action, it would be encouraging if the full results at those levels show an action density whose magnitude does not become larger.

One may also wonder what happens if terms of order higher than quartic are included in the potential. Since the tachyon terms in the CSFT potential alternate signs [10], the quintic term is positive and will help reduce the value of the action at the critical point. The coefficient of this coupling will be eventually needed as computations become more accurate. The sixtic term will have a destabilizing effect. Having survived the destabilizing effects of the quartic term, we can hope that those of the sixtic term will prove harmless. If, in general, even power terms do not have catastrophic effects, it may be better to work always with truncations of odd power.

3. The sigma model and the string field theory pictures

In this section we study the relations between the string field metric $h_{\mu\nu}$ and the ghost-dilaton d and the corresponding sigma model fields, the string metric $\tilde{h}_{\mu\nu}$ and dilaton Φ.

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These relations are needed to interpret the tachyon vacuum solution and to discuss the possible relation to the rolling solutions.

We begin by finding the precise linearized relations between the string field dilaton and the sigma model dilaton. The linearized relations confirm that the CSFT metric $h_{\mu\nu}$, which does not acquire an expectation value in the tachyon vacuum, coincides with the string metric of the sigma model, which does not change in the rolling solutions. Moreover, the relation (3.14), together with $h_{\mu\nu} = 0$, implies that our $d > 0$ in the tachyon vacuum corresponds to $\Phi > 0$, thus larger string coupling. This is also consistent with what we obtained in the rolling solutions.

Our discussion of the linearized relations also allows us to examine the various vertex operators associated with the various dilaton fields used in the literature (section 3.2.). In section 3.3 we examine the nonlinear relations between the CSFT tachyon and dilaton and the effective field theory ones. We work at zero momentum and up to quadratic order. Finally, in section 3.4, we present evidence that CSFT can describe arbitrarily large dilaton deformations.

3.1 Relating sigma model fields and string fields

Consider first the effective action (1.5), suggested by the conditions of conformal invariance of a sigma model with gravity, dilaton and tachyon background fields. If we set the tachyon to zero, this action reduces to the effective action for massless fields, in the conventions of [32]. In this action $g_{\mu\nu}$ is the string metric, Φ is the diffeomorphism invariant dilaton, and T, with potential $V(T) = -\frac{2}{\alpha'}T^2 + \cdots$, is the tachyon. In order to compare with the string field action we expand the effective action in powers of small fluctuations using

$$g_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu}\,, \tag{3.1}$$

where we use a tilde in the fluctuation to distinguish it from the metric fluctuation in the string field. The result is

$$\begin{aligned} S_\sigma = \frac{1}{2\kappa^2}\int d^Dx\Big(&\frac{1}{4}\tilde{h}_{\mu\nu}\partial^2\tilde{h}^{\mu\nu} - \frac{1}{4}\tilde{h}\partial^2\tilde{h} + \frac{1}{2}(\partial^\nu\tilde{h}_{\mu\nu})^2 + \frac{1}{2}\tilde{h}\partial_\mu\partial_\nu\tilde{h}^{\mu\nu} + 2\tilde{h}\,\partial^2\Phi - \\ &- 2\Phi\,\partial_\mu\partial_\nu\tilde{h}^{\mu\nu} - 4\Phi\,\partial^2\Phi - (\partial T)^2 + \frac{4}{\alpha'}T^2 + \tilde{h}^{\mu\nu}\partial_\mu T\partial_\nu T + \\ &+ \left(\frac{\tilde{h}}{2} - 2\Phi\right)(\partial T)^2 + \cdots\Big)\,, \end{aligned} \tag{3.2}$$

where we have kept cubic terms coupling the dilaton and metric to the tachyon. Such terms are needed to fix signs in the relations between the fields in the sigma model and the string fields.

Let us now consider the string field action. The string field needed to describe the tachyon, the metric fluctuations, and the dilaton is

$$\begin{aligned} |\Psi\rangle = \int\frac{d^Dk}{(2\pi)^D}\Big(&t(k)\,c_1\bar{c}_1 - \frac{1}{2}h_{\mu\nu}(k)\alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}c_1\bar{c}_1 + d(k)(c_1c_{-1} - \bar{c}_1\bar{c}_{-1}) + \\ &+ i\sqrt{\frac{\alpha'}{2}}B_\mu(k)c_0^+(c_1\alpha^\mu_{-1} - \bar{c}_1\bar{\alpha}^\mu_{-1})\Big)|k\rangle\,. \end{aligned} \tag{3.3}$$

Here $t(k)$ is the tachyon, $h_{\mu\nu}(k) = h_{\nu\mu}(k)$ is a metric fluctuation, $d(k)$ is the ghost-dilaton, and $B_\mu(k)$ is an auxiliary field. The sign and coefficient of $h_{\mu\nu}$ have been chosen for future convenience. The linearized gauge transformations of the component fields can be obtained from $\delta|\Psi\rangle = Q_B|\Lambda\rangle$ with

$$|\Lambda\rangle = \frac{i}{\sqrt{2\alpha'}}\,\epsilon_\mu(c_1\alpha^\mu_{-1} - \bar{c}_1\bar{\alpha}^\mu_{-1})|p\rangle\,. \tag{3.4}$$

The resulting coordinate-space gauge transformations are:

$$\delta h_{\mu\nu} = \partial_\nu\epsilon_\mu + \partial_\mu\epsilon_\nu\,, \qquad \delta d = -\frac{1}{2}\,\partial\cdot\epsilon\,, \qquad \delta B_\mu = -\frac{1}{2}\partial^2\epsilon_\mu\,, \qquad \delta\, t = 0\,. \tag{3.5}$$

We now calculate the quadratic part of the closed string field action, finding

$$\begin{aligned} S^{(2)} &= -\frac{1}{\kappa^2\alpha'}\,\langle\Psi|c_0^- Q_B|\Psi\rangle\,,\\ &= \frac{1}{2\kappa^2}\int d^Dx\Big(\frac{1}{4}h_{\mu\nu}\partial^2h^{\mu\nu} - 2d\,\partial^2 d\, - 2B_\mu(\partial_\nu h^{\mu\nu} + 2\partial^\mu d\,) - 2B^2 - (\partial t)^2 + \frac{4}{\alpha'}t^2\Big)\,,\\ &= \frac{1}{2\kappa^2}\int d^Dx\Big(\frac{1}{4}h_{\mu\nu}\partial^2h^{\mu\nu} + \frac{1}{2}(\partial^\nu h_{\mu\nu})^2 - 4d\,\partial^2 d - 2d\,\partial_\mu\partial_\nu h^{\mu\nu} - (\partial t)^2 + \frac{4}{\alpha'}t^2\Big)\,. \end{aligned} \tag{3.6}$$

In the last step we eliminated the auxiliary field B_μ using its algebraic equation of motion.

The gauge transformations (3.5) imply that the linear combination $d + \frac{h}{4}$ is gauge invariant. It follows that the sigma model dilaton must take the form

$$\lambda\,\Phi = d + \frac{h}{4}\,, \tag{3.7}$$

where λ is a number to be determined. Using (3.7) to eliminate the ghost-dilaton d from the action (3.6) we find

$$\begin{aligned} S^{(2)} = \frac{1}{2\kappa^2}\int d^Dx\Big(&\frac{1}{4}h_{\mu\nu}\partial^2h^{\mu\nu} - \frac{1}{4}h\partial^2 h + \frac{1}{2}(\partial^\nu h_{\mu\nu})^2 + \frac{1}{2}h\partial_\mu\partial_\nu h^{\mu\nu} +\\ &+ 2\lambda\, h\partial^2\Phi - 2\lambda\,\Phi\,\partial_\mu\partial_\nu h^{\mu\nu} - 4\lambda^2\,\Phi\partial^2\Phi - (\partial t)^2 + \frac{4}{\alpha'}t^2\Big)\,. \end{aligned} \tag{3.8}$$

We also use the string field theory to calculate the on-shell coupling of $h_{\mu\nu}$ to two tachyons. This coupling arises from the term

$$S^{(3)} = -\frac{1}{\alpha'\kappa^2}\langle\,\mathcal{T}\,,\mathcal{H}\,,\mathcal{T}\rangle\,, \tag{3.9}$$

where $\mathcal{T}$ and $\mathcal{H}$ denote the parts of the string field (3.3) that contain $t(k)$ and $h_{\mu\nu}(k)$, respectively. We thus have

$$S^{(3)} = \frac{1}{2\alpha'\kappa^2}\Big(\prod_{i=1}^{3}\int\frac{d^Dk_i}{(2\pi)^D}\Big)\big\langle c_1\bar{c}_1e^{ik_1\cdot X}, c_1\bar{c}_1\alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}e^{ik_2\cdot X}, c_1\bar{c}_1e^{ik_3\cdot X}\big\rangle\, t(k_1)t(k_3)h_{\mu\nu}(k_2)\,. \tag{3.10}$$

The on-shell evaluation is readily carried out using $k^\mu h_{\mu\nu}(k)=0$. We obtain

$$S^{(3)}=-\frac{1}{2\kappa^2}\int\frac{d^Dk_1}{(2\pi)^D}\frac{d^Dk_3}{(2\pi)^D}\,k_1^\mu k_3^\nu\,t(k_1)t(k_3)h_{\mu\nu}(-k_1-k_3)=\frac{1}{2\kappa^2}\int d^Dx\,h^{\mu\nu}\partial_\mu t\partial_\nu t\,. \tag{3.11}$$

Combining this result with (3.8) we obtain the closed string field theory action

$$\begin{aligned}S_{csft}=\frac{1}{2\kappa^2}\int d^Dx\Big(&\frac{1}{4}h_{\mu\nu}\partial^2h^{\mu\nu}-\frac{1}{4}h\partial^2h+\frac{1}{2}(\partial^\nu h_{\mu\nu})^2+\frac{1}{2}h\partial_\mu\partial_\nu h^{\mu\nu}+\\&+2\lambda\,h\partial^2\Phi-2\lambda\,\Phi\partial_\mu\partial_\nu h^{\mu\nu}-4\lambda^2\,\Phi\partial^2\Phi-\\&-(\partial t)^2+\frac{4}{\alpha'}t^2+h^{\mu\nu}\partial_\mu t\partial_\nu t+\cdots\Big)\,.\end{aligned} \tag{3.12}$$

We are finally in a position to identify the sigma model action (3.2) and the string field action (3.12). Comparing the quadratic terms in $\tilde{h}_{\mu\nu}$ and those in $h_{\mu\nu}$ we see that $\tilde{h}_{\mu\nu}=\pm h_{\mu\nu}$. We also note that $T=\pm t$. The coupling $\tilde{h}^{\mu\nu}\partial_\mu T\partial_\nu T$ in (3.2) coincides with the corresponding coupling in (3.12) if and only if

$$\tilde{h}_{\mu\nu}=h_{\mu\nu}\,. \tag{3.13}$$

This simple equality justifies the multiplicative factor of $(-1/2)$ introduced for $h_{\mu\nu}$ in the string field (3.3). The string field $h_{\mu\nu}$ so normalized is the fluctuation of the string metric. Comparing the couplings of metric and dilaton in both actions we also conclude that $\lambda=+1$ and, therefore, equation (3.7) gives

$$\Phi=d+\frac{h}{4}\,. \tag{3.14}$$

This expresses the sigma model dilaton Φ in terms of the string field metric trace and the ghost dilaton d. It is important to note that when we give a positive expectation value to d (and no expectation value to h) we are increasing the value of Φ and therefore increasing the value of the string coupling.

3.2 The many faces of the dilaton

Equipped with the precise relations between string fields and sigma-model fields we digress on the various dilaton fields used in the literature. Of particular interest are the corresponding vertex operators, which are determined by the CFT states that multiply the component fields in the closed string field.

We introduce the states

$$|\mathcal{O}^{\mu\nu}(p)\rangle=-\frac{1}{4}(\alpha_{-1}^\mu\bar{\alpha}_{-1}^\nu+\alpha_{-1}^\nu\bar{\alpha}_{-1}^\mu)|p\rangle\,,\qquad|\mathcal{O}^d(p)\rangle=(c_1c_{-1}-\bar{c}_1\bar{c}_{-1})|p\rangle\,. \tag{3.15}$$

The corresponding vertex operators are

$$\mathcal{O}^{\mu\nu}(p)=\frac{1}{2\alpha'}(\partial X^\mu\bar{\partial}X^\nu+\partial X^\nu\bar{\partial}X^\mu)\,e^{ipX}\,,\qquad\mathcal{O}^d(p)=\frac{1}{2}(c\partial^2c-\bar{c}\bar{\partial}^2c)\,e^{ipX}\,. \tag{3.16}$$

Working for fixed momentum, the string field (3.3) restricted to metric and dilaton fluctuations is

$$|\Psi\rangle = h_{\mu\nu}\,|\mathcal{O}^{\mu\nu}\rangle + d\,|\mathcal{O}^{d}\rangle\,. \tag{3.17}$$

This equation states that $\mathcal{O}^d$ is the vertex operator associated with the ghost-dilaton field d. An excitation by this vertex operator does not change the metric $h_{\mu\nu}$. Our transformation to a gauge invariant dilaton gives

$$\Phi = d + \frac{1}{4}\,h\,, \qquad \tilde{h}_{\mu\nu} = h_{\mu\nu}\,. \tag{3.18}$$

Here $\tilde{h}_{\mu\nu}$ is the fluctuation of the string metric. Inverting these relations

$$d = \Phi - \frac{1}{4}\,\tilde{h}\,, \qquad h_{\mu\nu} = \tilde{h}_{\mu\nu}\,. \tag{3.19}$$

Subtituting into the string field (3.17) we obtain

$$|\Psi\rangle = \tilde{h}_{\mu\nu}\Big(|\mathcal{O}^{\mu\nu}\rangle - \frac{1}{4}\,\eta^{\mu\nu}|\mathcal{O}^{d}\rangle\Big) + \Phi\,|\mathcal{O}^{d}\rangle\,. \tag{3.20}$$

It is interesting to note that $\mathcal{O}^d$ is the vertex operator associated with a variation of the gauge-invariant dilaton Φ and no variation of the string metric. On the other hand, $\mathcal{O}^{\mu\nu} - \frac{1}{4}\,\eta^{\mu\nu}\,\mathcal{O}^d$ varies the string metric and does not vary the gauge-invariant dilaton (although it varies the ghost-dilaton).

Finally, we consider the formulation that uses the Einstein metric $g^E_{\mu\nu}$ and the dilaton Φ. The field redefinition is

$$g^E_{\mu\nu} = \exp(2\omega)\,g_{\mu\nu}\,, \qquad \text{with } \omega = -\frac{2}{D-2}\,\Phi\,. \tag{3.21}$$

Expanding in fluctuation fields we obtain

$$h^E_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{4}{D-2}\,\eta_{\mu\nu}\,\Phi\,. \tag{3.22}$$

Solving for d and h_μ in terms of Φ and $h^E_{\mu\nu}$ we get

$$d = -\frac{2}{D-2}\,\Phi - \frac{1}{4}h^E\,, \qquad h_{\mu\nu} = h^E_{\mu\nu} + \frac{4}{D-2}\,\eta_{\mu\nu}\,\Phi\,. \tag{3.23}$$

Substituting into the string field (3.17) we obtain

$$|\Psi\rangle = h^E_{\mu\nu}\Big(|\mathcal{O}^{\mu\nu}\rangle - \frac{1}{4}\,\eta^{\mu\nu}|\mathcal{O}^{d}\rangle\Big) + \frac{2}{D-2}\,\Phi\,\Big(2\eta_{\mu\nu}|\mathcal{O}^{\mu\nu}\rangle - |\mathcal{O}^{d}\rangle\Big)\,. \tag{3.24}$$

Interestingly, the vertex operator that varies the Einstein metric (without variation of the dilaton) is the same as that for the string metric (see (3.20)). It is the dilaton operator that changes this time. The vertex operator

$$\mathcal{D} = 2\eta_{\mu\nu}\mathcal{O}^{\mu\nu} - \mathcal{O}^d = \Big(\frac{2}{\alpha'}\,\partial X\cdot\bar{\partial}X - \frac{1}{2}(c\partial^2 c - \bar{c}\bar{\partial}^2 c)\Big)e^{ipX}\,, \tag{3.25}$$

varies the dilaton without varying the Einstein metric. This is the dilaton vertex operator used almost exclusively in the early literature — it is naturally associated with the Einstein metric. The corresponding state $|\mathcal{D}(p)\rangle$ has a particularly nice property: it is annihilated by the BRST operator when $p^2 = 0$. Indeed,

$$Q_B\,|\mathcal{D}(p)\rangle = \frac{\alpha'}{2} p^2 c_0^+ |\mathcal{D}(p)\rangle\,. \tag{3.26}$$

The dilaton $\mathcal{D}$ is in fact the unique linear combination of the matter and ghost dilatons that has this property. For other combinations, terms linear in the momentum p (such as $(p\cdot\alpha_{-1})c_1\,\bar{c}_1\bar{c}_{-1}|p\rangle$), survive.

3.3 Relating the sigma model and string field dilaton and tachyon

The closed string theory potential V, as read from the effective action (1.5) is

$$\kappa^2 V = e^{-2\Phi}\big(\,V(T) + \cdots\big)\,, \qquad \text{with } V(T) = -T^2 + \cdots\,. \tag{3.27}$$

Here Φ and T are the zero momentum dilaton and tachyon fields in the effective field theory. The purpose of this section is to discuss the relation between Φ and T and the corresponding string fields d and t, both sets at zero-momentum. To do this we must consider the effective potential for d and t calculated in string field theory. We only have the potential itself. Collecting our previous results, we write

$$\begin{aligned}\kappa^2 V \;=\; & -t^2 + 1.6018\,t^3 - 3.0172\,t^4 +\; 3.8721\,t^3 d + (-0.8438\,t + 1.3682\,t^2)\,d^2 - \\ & -0.9528\,t\,d^3 - 0.1056\,d^4\,.\end{aligned} \tag{3.28}$$

The contributions from massive fields affect quartic and higher order terms. In our setup, the relevant terms arise when we eliminate the level-four massive fields using their kinetic terms in (2.25) and their linear couplings to t^2 in (2.23), to td in (2.24), and to d^2 in (2.26). We find

$$\Delta V = -\frac{6241}{186624}\,d^4 + \frac{25329}{16384}\,d^2\,t^2 - \frac{1896129}{4194304}\,t^4 \simeq -0.0334\,d^4 + 1.5460\,d^2 t^2 - 0.4521\,t^4\,. \tag{3.29}$$

It follows that the effective potential for the tachyon and the dilaton, calculated up to terms quartic in the fields and including massive fields of level four only, is given by:

$$\begin{aligned}\kappa^2 V_{\text{eff}} \;=\; & -t^2 + 1.6018\,t^3 - 3.4693\,t^4 + 3.8721\,t^3 d + (-0.8438\,t + 2.9142\,t^2)\,d^2 - \\ & -0.9528\,t\,d^3 - 0.1390\,d^4 + \cdots\,.\end{aligned} \tag{3.30}$$

The dots represent quintic and higher terms, which receive contributions both from elementary interactions and some integration of massive fields. We write, more generically

$$\begin{aligned}\kappa^2 V_{\text{eff}} \;=\; & -t^2 + a_{3,0}t^3 + a_{4,0}\,t^4 + a_{3,1}\,t^3 d + (a_{1,2}\,t + a_{2,2}\,t^2)\,d^2 + \\ & + a_{1,3}\,t\,d^3 + a_{0,4}\,d^4 \;+ \cdots\,.\end{aligned} \tag{3.31}$$

The values of the coefficients $a_{i,j}$ can be read comparing this equation with (3.30).

There are two facts about V_{eff} that make it clear it is not in the form of a ghost-dilaton exponential times a tachyon potential. First, it does not have a term of the form $t^2 d$ that would arise from the tachyon mass term and the expansion of the exponential. Second, it contains a term linear in the tachyon; those terms should be absent since the tachyon potential does not have a linear term. Nontrivial field redefinitions are necessary to relate string fields and sigma model fields.

To linearized order the fields are the same, so we write relations of the form:

$$\begin{aligned} t &= T + \alpha_1 T\Phi + \alpha_2 \Phi^2 + \cdots , \\ d &= \Phi + \beta_0 T^2 + \beta_1 T\Phi + \beta_2 \Phi^2 + \cdots , \end{aligned} \tag{3.32}$$

where the dots indicate terms of higher order in the sigma model fields. We found no *need* for a T^2 term in the redefinition of tachyon field, such a term would change the cubic and quartic self-couplings of the tachyon in $V(T)$. Since d gives rise to pure tachyon terms that are quadratic or higher, only at quintic and higher order in T will $V(T)$ differ from the potential obtained by replacing $t \to T$ in the first line of (3.30). We thus expect that after the field redefinition (3.30) becomes

$$\kappa^2 V = e^{-2\Phi}\big(- T^2 + 1.6018\, T^3 - 3.4693\, T^4 + \ldots \big) , \tag{3.33}$$

at least to quartic order in the fields. We now plug the substitutions (3.32) into the potential (3.30) and compare with (3.33). A number of conditions emerge.

- In order to get the requisite $T^2\Phi$ term we need $\alpha_1 = -1$.
- In order to have a vanishing $T\Phi^2$ term $\alpha_2 = \frac{1}{2} a_{1,2}$ must be half the coefficient of td^2 in (3.30).
- Getting the correct $T^3\Phi$ coupling then fixes $\beta_0 = (a_{3,0} - a_{3,1})/(2a_{1,2})$.
- Getting the correct value of $T^2\Phi^2$ fixes $\beta_1 = -(1 + \frac{3}{2} a_{3,0} a_{1,2} + a_{2,2})/(2a_{1,2})$. The vanishing of $T\Phi^3$ fixes $\beta_2 = -a_{1,3}/(2a_{1,2})$. All coefficients in (3.32) are now fixed.
- The coefficient of Φ^4, which should be zero, turns out to be $(a_{0,4} + \frac{1}{4} a_{1,2}^2) \simeq 0.0389$, which is small, but does not vanish.

Our inability to adjust the coefficient of Φ^4 was to be expected. The potential (3.30) contains the terms $-t^2 + a_{1,2}\, td^2 + a_{0,4} d^4$ and, to this order, integrating out the tachyon gives an effective dilaton quartic term of $(a_{0,4} + \frac{1}{4} a_{1,2}^2)$. With the contribution of the massive fields beyond level four this coefficient in the dilaton effective potential would vanish. This is, in fact, the statement that was verified in [16]. It follows that we need not worry that the quartic term in Φ do not vanish exactly. Following the steps detailed before we find

$$\begin{aligned} t &= T - T\,\Phi - \,0.4219\,\Phi^2 + \cdots , \\ d &= \Phi + 1.3453\, T^2 + 1.1180\, T\,\Phi - \,0.5646\,\Phi^2 + \cdots . \end{aligned} \tag{3.34}$$

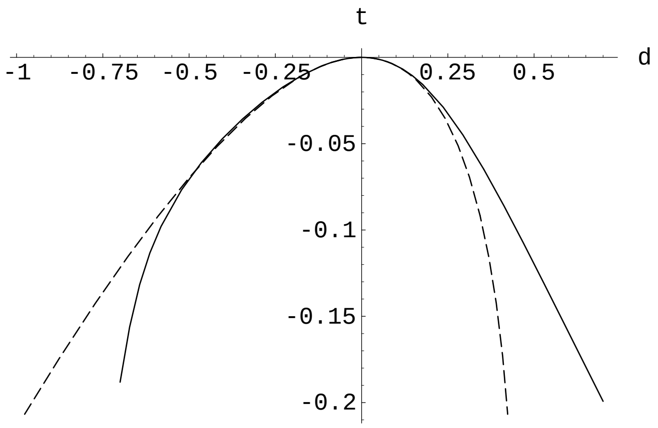

Figure 2: The solid line is the dilaton marginal direction defined by the set of points $(d, t(d))$ where $t(d)$ is the expectation value of t obtained solving the tachyon equation of motion for the given d. The dashed line represents the direction along the sigma model dilaton Φ (thus $T = 0$). It is obtained by setting $T = 0$ in equation (3.34). The two lines agree well even reasonably far from the origin.

In string field theory the dilaton deformation is represented in the (d, t) plane by the curve $(d, t(d))$, where $t(d)$ is the expectation value of the tachyon when the dilaton is set equal to d. This curve, calculated using the action (3.30), is shown as a solid line in figure 2. On the other hand, it is clear that Φ (with $T = 0$) defines the marginal direction in the effective field theory. Setting $T = 0$ in (3.34) we find the pair $(d(\Phi), t(\Phi))$, which must be a parameterization of the flat direction in terms of Φ. This curve is shown as a dashed line in figure 2. It is a good consistency check that these two curves agree well with each other over a significant fraction of the plot.

3.4 Dilaton deformations

In ref. [16] we computed the effective dilaton potential that arises when we integrate out the tachyon from a potential that includes only quadratic and cubic terms. We found that the domain of definition of this potential is the full real d line. This happens because the (marginal) branch $t(d)$ that gives the expectation value of t for a given value of d is well defined for all values of d. In this section we extend this computation by including higher level fields and higher order interactions. As we will demonstrate, it appears plausible that the domain of definition for the effective dilaton potential remains $d \in (-\infty, \infty)$.

The marginal branch is easily identified for small values of the dilaton: as the dilaton expectation value goes to zero all expectation values go to zero. For large enough values of the dilaton the marginal branch may cease to exist, or it may meet another solution branch. If so, we obtain limits on the value of d. Since the dilaton effective potential is supposed to be flat in the limit of high level, we propose the following criterion. If we encounter a limit value of d, this value is deemed reliable only if the dilaton potential at this point is not very large. A large value for the potential indicates that the calculation is not reliable because the same terms that are needed to make the potential small could well affect the limit value. In open string field theory a reliable limit value was obtained for the Wilson line parameter: at the limit point the potential energy density was a relatively small fraction of the D-brane energy density. The purely cubic potential for t gives a critical point with $\kappa^2 V \sim -0.05774$. We define $\mathcal{R}(d) \equiv \frac{|\kappa^2 V(d)|}{0.05774}$, where $V(d)$ is the effective dilaton potential. A critical value of d for which $\mathcal{R} > 1$ will be considered unreliable.

We start with cubic potentials and then include the elementary quartic interactions level by level. With cubic potentials, the effective dilaton potential is invariant under $d \to -d$. With $\mathbb{V}_4^{(3)}$ dilaton deformations can be arbitrarily large [16]. We then find

- The dilaton potential derived from $\mathbb{V}_8^{(3)}$ is defined for $|d| \leq 624$. This is plausible since, at this level, the equations of motion for the level-four fields are linear.

- The dilaton potential derived from $\mathbb{V}_{12}^{(3)}$ is defined for $|d| \leq 1.71$. Since $\mathcal{R}(\pm 1.71) = 42.4$, there is no reliable limit value.

- The dilaton potential derived from $\mathbb{V}_0^{(4)}$ is defined for $|d| \leq 4.67$, where $\mathcal{R}(\pm 4.67) = 49.5$. The large value of $\mathcal{R}$ indicates that there is no evidence of a limit value.

- The dilaton potential derived from $\mathbb{V}_2^{(4)}$ is not invariant under $d \to -d$. We find a range $d \in (-\infty, 3.124)$. Although $\mathcal{R}(3.124) = 0.387$, the potential has a maximum with $\mathcal{R} = 3.325$ at $d = 1.92$. This fact makes the limit point $d = 3.124$ unreliable.

- The dilaton potential derived from $\mathbb{V}_4^{(4)}$, the highest level potential we have computed fully, is regular for $d \in (-2.643,\ 6.415)$. Since $\mathcal{R}(6.415) = 1502.4$ and $\mathcal{R}(-2.643) = 89.2$, there is no branch cut in the reliable region.

We have also computed the higher level quartic interactions td^3 and d^4. We have checked that $\mathbb{V}_4^{(4)}$, supplemented by those interactions does not lead to branch cuts in the potential for the dilaton. This result, however, is not conclusive. Additional interactions must be included at level six (the level of td^3) and at level eight (the level of d^4).

We tested in [16] that cubic and quartic interactions combine to give a vanishing quartic term in the dilaton effective potential. We can ask if the potential for the dilaton becomes flatter as the level of the calculation is increased. We find that it roughly does, but the major changes in the potential are due to the elementary quartic term in the dilaton. For the cubic vertex, the interactions of the type $d^2 M$, with M massive give rise to terms quartic on the dilaton. Other cubic couplings that do not involve the dilaton typically induce d^6 (and higher order) terms, which play a secondary role in flattening the potential if the quartic terms have not cancelled completely. Therefore, the potentials that arise from $\mathbb{V}_8^{(3)}$, $\mathbb{V}_{10}^{(3)}$ and $\mathbb{V}_{12}^{(3)}$ (without the contribution from level six massive fields) have no obvious difference. The potentials obtained at various levels are shown in figure 3. The dashed line arises from $\mathbb{V}_4^{(3)}$, the solid line arises from $\mathbb{V}_8^{(3)}$, and the thick line arises from $\mathbb{V}_8^{(4)}$.

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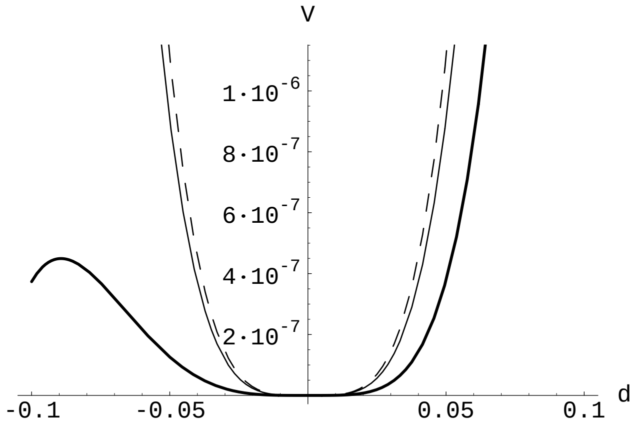

Figure 3: Dilaton effective potential. The dashed line arises from $\mathbb{V}_4^{(3)}$, the solid line arises from $\mathbb{V}_8^{(3)}$, and the thick line arises from $\mathbb{V}_8^{(4)}$.

4. Conclusions

In this paper we have presented some calculations that suggest the existence of a tachyon vacuum for the bulk closed string tachyon of bosonic string theory. We have discussed the physical interpretation using the effective field theory both to suggest the value of the action density at the critical point (zero!) and to obtain rolling solutions [17] that seem consistent with the interpretation of the tachyon vacuum as a state in which there are no closed string states.

The numerical evidence presented is still far from conclusive. A critical point seems to exist and appears to be robust, but it is not all that clear what will happen when the accuracy of the computation is increased. If the action density at the critical point goes to zero it may indeed define a new and nontrivial tachyon vacuum. Conceivably, however, the critical point could approach the perturbative vacuum, in which case there would be no evidence for a new vacuum. Alternatively, if the action density at the critical point remains finite, we would have no interpretation for the result.

Let us consider some additional indirect arguments that support the existence of a closed string tachyon vacuum. The first one arises from the existence of sub-critical bosonic string theories. The evidence in string theory is that most string theories are related by compactifications and/or deformations. It seems very likely that non-critical string theories are also related to critical string theory. It should then be possible to obtain a non-critical string theory as a solution of critical string theory. Certainly the view that $D = 2$ bosonic string theory is a ground state of the bosonic string has been held as likely [34]. In non-critical string theory the number of space dimensions is reduced (at the expense of a linear dilaton background). The analogy with lower-dimensional D-branes in open string theory seems apt: the branes are solitons of the open string field theory tachyon in which far away from the branes the tachyon sits at the vacuum. It seems plausible that non-critical string theories are solitonic solutions of the *closed* string theory tachyon. As sketched in figure 4, far away along the coordinates transverse to the non-critical world-volume, the

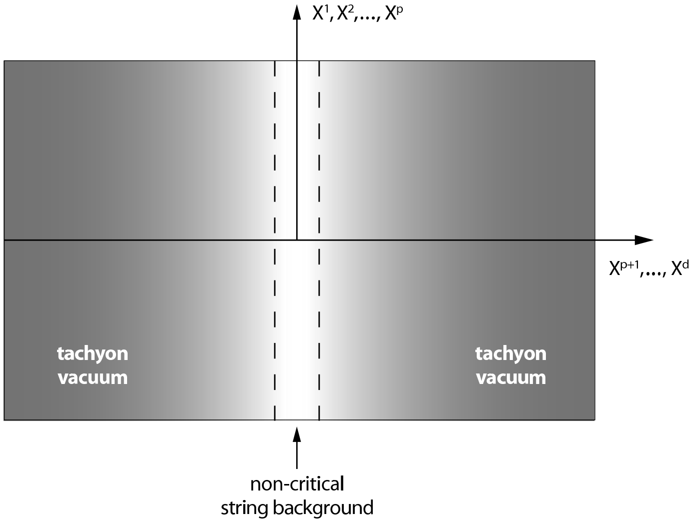

Figure 4: A non-critical $(p+1)$-dimensional string theory would correspont to a solitonic solution of critical string theory in which, far away from the reduced space, the fields approach the values of the closed string tachyon vacuum.

background would approach the closed string tachyon vacuum. The universality of the tachyon vacuum would imply that a noncritical string theory could be further reduced using the same background configuration used to reduce the original critical theory.

In fact, in the p-adic open/closed string theory lump solutions of the closed string sector appear to describe spacetimes of lower dimensionality, as explained by Moeller and Schnabl [30]. Indeed, far away from the lump the open string tachyon must be at its vacuum and therefore there are no D-brane solutions with more space dimensions than those of the lump. Away from the lump the closed string tachyon is at its vacuum, and no linearized solutions of the equations of motion exist.

A suggestive argument for zero action at the tachyon vacuum follows from the sigma model approach. As discussed by Tseytlin [27], it seems likely that the closed string effective action for the spacetime background fields may be written in terms of the partition function Z of the two-dimensional sigma model as well as derivatives thereof (this does work for open strings [29]). The conventional coupling of the world-sheet area to the tachyon T results in a partition function and an effective action with a prefactor of e^{-T}. Thus one expects a tachyon potential of the form $e^{-T}g(T)$ where g is a polynomial that begins with a negative quadratic term[4]. In this case, for a tachyon vacuum at $T \to \infty$ the action goes to zero.

The computations and the discussion presented in this paper have led to a set of testable conjectures concerning the vacuum of the bulk closed string tachyon of bosonic string theory. It seems likely that additional computations, using both string field theory, effective field theory, and conformal field theory will help test these ideas in the near future.

[4]In [27], a tachyon potential of the form $-T^2e^{-T}$ is considered. Complications in fixing the kinetic terms made it unclear if $T = \infty$ was a point in the configuration space (see the discussion below eq. (4.13)) of [27]. For additional comments on the possible form of the tachyon potential, see Andreev [28].

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A. Quartic computations

A.1 The setup

We normalize correlators using $\langle 0|c_{-1}\bar{c}_{-1}c_0^- c_0^+ c_1\bar{c}_1|0\rangle = 1$ with $c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$. All states in this paper have zero momentum. For convenience, all spacetime coordinates have been compactified and the volume of spacetime is equal to one. To use results from open string field theory, we note that

$$\langle c(z_1)c(z_2)c(z_3)\,\bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3)\rangle = -2\langle c(z_1)c(z_2)c(z_3)\rangle_o \cdot \langle \bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3)\rangle_o\,, \qquad \text{(A.1)}$$

since open string field theory uses $\langle c(z_1)c(z_2)c(z_3)\rangle_o = (z_1 - z_2)(z_1 - z_3)(z_2 - z_3)$. Then:

$$\langle c_1\bar{c}_1, c_1\bar{c}_1, c_1\bar{c}_1\rangle = 2\cdot\langle c_1, c_1, c_1\rangle_o \ \cdot \langle \bar{c}_1, \bar{c}_1, \bar{c}_1\rangle_o = 2\cdot\mathcal{R}^3\cdot\mathcal{R}^3 = 2\mathcal{R}^6\,, \qquad \text{(A.2)}$$

where $\mathcal{R} \equiv 1/\rho = 3\sqrt{3}/4 \simeq 1.2990\,,$ and ρ is the mapping radius of the disks in the three-string vertex.

To construct four-string amplitudes we use antighost insertions [7, 10]

$$\mathcal{B} = \sum_{I=1}^{4}\sum_{m=-1}^{\infty}(B^I_m\, b^J_m + \overline{C^I_m}\,\bar{b}^I_m)\,, \qquad \mathcal{B}^\star = \sum_{I=1}^{4}\sum_{m=-1}^{\infty}(C^I_m\, b^I_m + \overline{B^I_m}\,\bar{b}^I_m)\,, \qquad \text{(A.3)}$$

where $\mathcal{B}^\star$ is the $\star$-conjugate of $\mathcal{B}$. The multilinear function in string field theory is

$$\{\Psi_1, \Psi_2, \Psi_3, \Psi_4\} \equiv \frac{1}{\pi}\int_{\mathcal{V}_{0,4}} dx\wedge dy\,\langle\Sigma|\,\mathcal{B}\,\mathcal{B}^\star\,|\Psi_1\rangle|\Psi_2\rangle|\Psi_3\rangle|\Psi_4\rangle\,. \qquad \text{(A.4)}$$

The first, second, third, and fourth states are inserted at $0, 1, \xi = x+iy$, and ∞, respectively. Operationally, the fourth state is inserted at $t = 0$ with $z = 1/t$, where z is the global uniformizer. For further details and explanations the reader should consult [16]. We record that

$$\begin{aligned}
B^J_{-1} &= \frac{\delta_{3J}}{\rho_3}\,, \qquad C^J_{-1} = 0\,,\\
B^I_1 &= \rho_I\partial\beta_I + \frac{1}{2}\rho_3\varepsilon_3\delta_{I3}\,, \qquad C^I_1 = \rho_I\bar{\partial}\beta_I\,,\\
B^I_2 &= \frac{1}{6}\rho_I^2\partial(2\beta_I^2 - \varepsilon_I) + \rho_I^2(-4\delta_I - 2\varepsilon_I\beta_I + 8\beta_I^3)\delta_{3I}\,, \qquad C^I_2 = \tfrac{1}{6}\rho_I^2\bar{\partial}(2\beta_I^2 - \varepsilon_I)\,.
\end{aligned} \qquad \text{(A.5)}$$

Here $\bar{\partial} \equiv \partial/\partial\bar{\xi}$ and $\partial \equiv \partial/\partial\xi$. Since our string fields are annihilated both by b_0 and $\bar{b}_0$, the coefficients B^I_0 and C^I_0 are not needed. Taking note of the vanishing coefficients, we see that for states in the Siegel gauge the antighost factor $\mathcal{B}$ is given by

$$\mathcal{B} = B^3_{-1}b^{(3)}_{-1} + \sum_{I=1}^{4}(B^I_1\, b^J_1 + \overline{C^I_1}\,\bar{b}^I_1) + \sum_{I=1}^{4}(B^I_2\, b^J_2 + \overline{C^I_2}\,\bar{b}^I_2) + \cdots\,. \qquad \text{(A.6)}$$

The Strebel quadratic differential on the surfaces determines:

$$\beta_1 = \frac{a}{2\xi} - \frac{1}{\xi} - 1\,, \qquad \beta_2 = \frac{a-2\xi}{2(1-\xi)}\,, \qquad \beta_3 = \frac{a-2}{2\xi(\xi-1)}\,, \qquad \beta_4 = \frac{a}{2} - 1 - \xi\,. \quad \text{(A.7)}$$

Here $a(\xi, \bar{\xi})$ is a function that determines the quadratic differential completely. We also have

$$\begin{aligned}
\varepsilon_1 &= 2 + \frac{1}{\xi}(a-2) + \frac{1}{\xi^2}\left(2 + a - \frac{5}{8}a^2\right),\\
\varepsilon_2 &= \frac{-5a^2 + 16\xi(\xi-3) + 8a\,(\xi+3)}{8\,(\xi-1)^2}\,,\\
\varepsilon_3 &= \frac{16 + 8a - 5a^2 + 24(a-2)\xi}{8\,\xi^2\,(\xi-1)^2}\,,\\
\varepsilon_4 &= 2 + a - \frac{5}{8}\,a^2 - 2\xi + a\,\xi + 2\xi^2\,.
\end{aligned} \quad \text{(A.8)}$$

The function $a(\xi)$ is known numerically to high accuracy for $\xi \in \mathcal{A}$, where $\mathcal{A}$ is a specific subspace of $\mathcal{V}_{0,4}$ described in detail in figures 3 and 6 of ref. [12]. The full space $\mathcal{V}_{0,4}$ is obtained by acting on $\mathcal{A}$ with the transformations generated by $\xi \to 1-\xi$ and $\xi \to 1/\xi$, together with complex conjugation $\xi \to \bar{\xi}$. In fact $\mathcal{V}_{0,4}$ contains twelve copies of $\mathcal{A}$. Let $f(\mathcal{A})$ denote the region obtained by mapping each point $\xi \in \mathcal{A}$ to $f(\xi)$. Then $\mathcal{V}_{0,4}$ is composed of the six regions

$$\mathcal{A}\,, \qquad \frac{1}{\mathcal{A}}\,, \qquad 1-\mathcal{A}\,, \qquad \frac{1}{1-\mathcal{A}}\,, \qquad 1-\frac{1}{\mathcal{A}}\,, \qquad \frac{\mathcal{A}}{1-\mathcal{A}}\,, \quad \text{(A.9)}$$

together with their complex conjugates. The values of a in these regions follow from the values of a on $\mathcal{A}$ via the relations

$$a(1-\xi) = 4 - a(\xi)\,, \qquad a\left(\frac{1}{\xi}\right) = \frac{a(\xi)}{\xi}\,, \qquad a(\bar{\xi}) = \overline{a(\xi)}\,. \quad \text{(A.10)}$$

For states of the form $|M_i\rangle = \mathcal{O}_i c_1 \bar{c}_1 |0\rangle$, where $\mathcal{O}_i$ is built with matter oscillator, one finds

$$\{M_1, M_2, M_3, M_4\} = -\frac{2}{\pi}\int_{\mathcal{V}_{0,4}} \frac{dx\wedge dy}{(\rho_1\rho_2\rho_3\rho_4)^2}\,\langle\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle\rangle_\xi\,. \quad \text{(A.11)}$$

Here $\langle\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle\rangle_\xi \equiv \langle h_1\circ\mathcal{O}_1\ h_2\circ\mathcal{O}_2\ h_3\circ\mathcal{O}_3\ h_4\circ\mathcal{O}_4\rangle_{\Sigma_\xi}$, where the right-hand side is a matter correlator computed after the local operators $\mathcal{O}_i$ have been mapped to the uniformizer.

A.2 Couplings of dilatons and tachyons

Elementary contribution to $t^3 d$. We insert the dilaton on the moving puncture to make the integration identical over each of the 12 regions of the moduli space. Since all the states inserted on the fixed punctures have ghost oscillators $c_1\bar{c}_1$, the antighost factor $\mathcal{B}\,\mathcal{B}^\star$ is only supported on the moving puncture:

$$\mathcal{B}\,\mathcal{B}^\star(c_1c_{-1} - \bar{c}_1\bar{c}_{-1})^{(3)}|0\rangle = -(B^3_{-1}C^3_1 + \overline{B^3_{-1}}\,\overline{C^3_1})|0\rangle = -(\bar{\partial}\beta_3 + \partial\bar{\beta}_3)|0\rangle\,. \quad \text{(A.12)}$$

There are no matter operators, thus the correlator just involves the ghosts:

$$\begin{aligned}\langle\Sigma_P|\mathcal{B}\,\mathcal{B}^\star|T\rangle|T\rangle|D\rangle|T\rangle &= -(\bar\partial\beta_3+\partial\bar\beta_3)\langle(c_1\bar c_1)^{(1)}(c_1\bar c_1)^{(2)}(c_1\bar c_1)^{(4)}\rangle \\ &= -(\bar\partial\beta_3+\partial\bar\beta_3)\frac{2}{(\rho_1\rho_2\rho_4)^2}\,. \end{aligned} \tag{A.13}$$

Using (A.4), the amplitude is:

$$\{T^3D\} = -\frac{24}{\pi}\int_{\mathcal{A}} dxdy(\bar\partial\beta_3+\partial\bar\beta_3)\frac{1}{(\rho_1\rho_2\rho_4)^2} = 23.2323\,. \tag{A.14}$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!}\{T^3D\}\,t^3d = 3.8721\,t^3d\,.$

Elementary contribution to t^2d^2. We insert the dilatons at $z_2 = 1$ and $z_3 = \xi$. The amplitude to be integrated is identical to the ghost part of the amplitude for the quartic interaction a^2d^2, as given in [16], eq. (4.9):

$$\langle\Sigma|\mathcal{B}\mathcal{B}^\star|T\rangle|D\rangle|D\rangle|T\rangle = \frac{2}{(\rho_1\rho_4)^2}\Big(\bar\partial\beta_2\partial(\xi\bar\beta_3)-\partial\beta_2\bar\partial(\bar\xi\bar\beta_3)+*\text{-conj}\Big)\,. \tag{A.15}$$

The four-point amplitude is then

$$\{T^2D^2\} = \frac{4}{\pi}\int_{\mathcal{V}_{0,4}}\frac{dxdy}{(\rho_1\rho_4)^2}\,\mathrm{Re}\,\Big(\bar\partial\beta_2\partial(\xi\bar\beta_3)-\partial\beta_2\bar\partial(\bar\xi\bar\beta_3)\Big)\,. \tag{A.16}$$

Since we have the same states on punctures one and four, and these punctures are exchanged by the transformation $z\to 1/z$, the integral over $\mathcal{A}$ gives the same contribution as the integral over $1/\mathcal{A}$. The conjugation properties of the amplitude also imply that $\overline{\mathcal{A}}$ contributes the same as $\mathcal{A}$. Consequently, the four regions $\mathcal{A}$, $1/\mathcal{A}$, $\overline{\mathcal{A}}$, and $1/\overline{\mathcal{A}}$ all give the same contribution. To get the full amplitude we must multiply the contributions of $\mathcal{A}$, of $1-\mathcal{A}$, and $1-1/\mathcal{A}$ by four:

$$\{T^2D^2\} = 4\cdot\frac{4}{\pi}\left[\int_{\mathcal{A}}+\int_{1-\mathcal{A}}+\int_{1-1/\mathcal{A}}\right]\frac{dxdy}{(\rho_1\rho_4)^2}\,\mathrm{Re}\,\Big(\bar\partial\beta_2\partial(\xi\bar\beta_3)-\partial\beta_2\bar\partial(\bar\xi\bar\beta_3)\Big)\,. \tag{A.17}$$

The transformation laws given in appendix B of [16] allow one to rewrite the second and third integrals as integrals over $\mathcal{A}$, where they can be easily evaluated. We find

$$\{T^2D^2\} = 4\cdot(-0.2410+0.4031+1.2065) = 5.4726\,. \tag{A.18}$$

The contribution to potential is $\kappa^2 V = \frac{6}{4!}\{T^2D^2\}\,t^2d^2 = 1.3682\,t^2d^2$.

Elementary contribution to td^3. The tachyon field is inserted at $z_3 = \xi$. We then have

$$\begin{aligned}
\mathcal{B}\,\mathcal{B}^{\star}(c_1\bar{c}_1)^{(3)}D^{(1)}D^{(2)}D^{(4)}|0\rangle \;=&\; \Big\{B^3_{-1}b^{(3)}_{-1} + \sum_{J\neq 3}\big(B^J_1 b^{(J)}_1 + \overline{C^J_1}\,\bar{b}^{(J)}_1\big)\Big\}\times \\
&\times\Big\{\overline{B^3_{-1}}\,\bar{b}^{(3)}_{-1} + \sum_{J\neq 3}\big(\overline{B^J_1}\,\bar{b}^{(J)}_1 + C^J_1 b^{(J)}_1\big)\Big\}\times \\
&\times(c_1\bar{c}_1)^{(3)}D^{(1)}D^{(2)}D^{(4)}|0\rangle \\
=&\sum_{I\neq J\neq K\neq 3}\Big(\frac{1}{2}B^3_{-1}C^I_1 D^{(J)}\,D^{(K)}\,c^{(I)}_1\bar{c}^{(3)}_1 + \\
&+B^I_1C^J_1(c_1\bar{c}_1)^{(3)}c^{(I)}_1c^{(J)}_1(\bar{c}_{-1}\bar{c}_1)^{(K)}\Big)|0\rangle - \star\text{-conj}\,.
\end{aligned} \tag{A.19}$$

Therefore, the correlator $\mathcal{C}_{td^3} = \langle\Sigma|\mathcal{B}\,\mathcal{B}^{\star}TD^3|0\rangle$ is:

$$\begin{aligned}
\mathcal{C}_{td^3} \;=& \sum_{I\neq J\neq K\neq 3}\Big\langle -B^3_{-1}C^I_1(\bar{c}_{-1}\bar{c}_1)^{(J)}(c_{-1}c_1)^{(K)}\,c^{(I)}_1\bar{c}^{(3)}_1 + \\
&+ B^I_1C^J_1(\bar{c}_{-1}\bar{c}_1)^{(K)}c^{(I)}_1c^{(J)}_1(c_1\bar{c}_1)^{(3)}\Big\rangle + *\text{-conj}\,.
\end{aligned}$$

Factorizing into holomorphic and antiholomorphic parts we get

$$\mathcal{C}_{td^3} = 2\sum_{I\neq J\neq K\neq 3}\Big(B^3_{-1}C^I_1B_{KI}(B_{J3})^* - \,B^I_1C^J_1D_{IJ}(B_{K3})^*\Big) + *\text{-conj}\,, \tag{A.20}$$

where $B_{IJ} \equiv \langle (c_{-1}c_1)^{(I)}, c^{(J)}_1\rangle$ was introduced and evaluated in [16], eqs. (4.18), (4.20), and (4.21). Additionally,

$$D_{IJ} \equiv \langle c^{(I)}_1, c^{(J)}_1, c^{(3)}_1\rangle = \frac{z_{IJ}z_{I3}z_{J3}}{\rho_I\rho_J\rho_3}\,, \qquad D_{I4} = -D_{4I} = \frac{z_{I3}}{\rho_I\rho_3\rho_4}\,, \quad I, J\neq 4\,. \tag{A.21}$$

The full amplitude is

$$\{TD^3\} = \frac{12}{\pi}\int_{\mathcal{A}} dxdy\,\mathcal{C}_{td^3} \;=\; -5.7168\,. \tag{A.22}$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!}\{TD^3\}\,td^3 = -0.9528\,td^3$.

A.3 Couplings of tachyon to massive fields

In all cases the massive field will be inserted on the moving puncture $z_3 = \xi$.

Elementary contribution to t^3f_1. With $F_1 \equiv c_{-1}\bar{c}_{-1}$ inserted at $z_3 = \xi$ we find:

$$\mathcal{B}\,\mathcal{B}^{\star}(c_{-1}\bar{c}_{-1})^{(3)}|0\rangle \;=\; (C^3_1\overline{C^3_1} - B^3_1\,\overline{B^3_1})|0\rangle. \tag{A.23}$$

$$\{T^3F_1\} \;=\; \frac{12}{\pi}\int_{\mathcal{A}} dxdy\,\frac{2}{(\rho_1\rho_2\rho_4)^2}(C^3_1\overline{C^3_1} - B^3_1\,\overline{B^3_1}) = -2.6261\,. \tag{A.24}$$

The contribution to the potential is: $\kappa^2 V = \frac{4}{4!}\{T^3F_1\}t^3f_1 = -0.4377\,t^3f_1$.

Elementary contribution to $t^3 f_2$. With $F_2 \equiv c_1\bar{c}_1 L_{-2}\bar{L}_{-2}$ at $z_3 = \xi$, the ghost part is that of the four-tachyon amplitude (eq. (3.34) of [16]). With $w = 0$ corresponding to $z = z_3$, and $S(z,w)$ denoting the Schwarzian derivative, the holomorphic matter correlator is:

$$\langle L^{(3)}_{-2}\rangle = \langle T^{(3)}(w=0)\rangle = \rho_3^2\langle T(z_3)\rangle + \frac{26}{12}S(z,w) = \frac{13}{6}\rho_3^2(2\beta_3^2 - \varepsilon_3)\,. \tag{A.25}$$

Therefore, the amplitude is

$$\{T^3F_2\} = -\frac{24}{\pi}\int_{\mathcal{A}} \frac{dxdy}{(\rho_1\rho_2\rho_3\rho_4)^2}\left|\frac{13}{6}\rho_3^2(2\beta_3^2-\varepsilon_3)\right|^2 = -337.571\,. \tag{A.26}$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!}\{T^3F_2\}t^3 f_2 = -56.262\, t^3 f_2$.

Elementary contribution to $t^3 f_3$. With $L_{-2}c_1\bar{c}_{-1}$ inserted at $z_3 = \xi$ we find

$$\mathcal{B}\,\mathcal{B}^\star(c_1\bar{c}_{-1})^{(3)}|0\rangle \;=\; -B^3_{-1}\overline{B^3_1}|0\rangle\,. \tag{A.27}$$

$$\begin{aligned}\mathcal{C}_{t^3 f_3} &\equiv \langle\Sigma|\mathcal{B}\,\mathcal{B}^\star TT(c_1\bar{c}_{-1})^{(3)}T|0\rangle\cdot\langle L^{(3)}_{-2}\rangle\\ &= -\frac{2B^3_{-1}\overline{B^3_1}}{(\rho_1\rho_2\rho_4)^2}\cdot\frac{13}{6}\rho_3^2(2\beta_3^2-\varepsilon_3)\,.\end{aligned} \tag{A.28}$$

With $F_3 \equiv L_{-2}c_1\bar{c}_{-1} + c_{-1}\bar{L}_{-2}\bar{c}_1$, the string amplitude relevant to $t^3 f_3$ is:

$$\{T^3\,F_3\} = \frac{12}{\pi}\int_{\mathcal{A}} dxdy\,(\mathcal{C}_{t^3 f_3} + \mathcal{C}^*_{t^3 f_3}) = 78.1432\,. \tag{A.29}$$

The contribution to the potential is: $\kappa^2 V = \frac{4}{4!}\{T^3F_3\}\,t^3 f_3 = 13.024\, t^3 f_3$.

Elementary contribution to $t^3 g_1$. With $b_{-2}c_1\,\bar{c}_{-2}\bar{c}_1$ at $z_3 = \xi$, one finds

$$\mathcal{B}\,\mathcal{B}^\star(b_{-2}c_1\bar{c}_{-2}\bar{c}_1)^{(3)}|0\rangle = \overline{C^3_2}\,\overline{B^3_{-1}}(c_1b_{-2})^{(3)}|0\rangle\,. \tag{A.30}$$

The state $c_1b_{-2}|0\rangle$ is created by the non-primary ghost current $j(z) = cb(z)$ by acting on the vacuum. For the ghost current

$$j(w) = j(z)\frac{dz}{dw} - \frac{3}{2}\frac{z''}{z'} \quad\to\quad j(w=0) = \rho_3(j(z_3) - 3\beta_3)\,. \tag{A.31}$$

We thus have the correlator:

$$\begin{aligned}\mathcal{C}_{t^3 g_1} &\equiv \langle\Sigma|\mathcal{B}\,\mathcal{B}^\star TT(b_{-2}c_1\,\bar{c}_{-2}\bar{c}_1)^{(3)}T|0\rangle\\ &= \overline{C^3_2}\,\overline{B^3_{-1}}\frac{1}{(\rho_1\rho_2\rho_4)^2}\Big\langle c\bar{c}(0)\,c\bar{c}(1)\rho_3(j(z_3)-3\beta_3)c\bar{c}(t=0)\Big\rangle\\ &= \overline{C^3_2}\,\overline{B^3_{-1}}\frac{\rho_3}{(\rho_1\rho_2\rho_4)^2}\cdot 2\left(\frac{1}{\xi}+\frac{1}{\xi-1}-3\beta_3\right).\end{aligned} \tag{A.32}$$

With $G_1 \equiv b_{-2}c_1\,\bar{c}_{-2}\bar{c}_1 - c_{-2}c_1\bar{b}_{-2}\bar{c}_1$, the amplitude relevant for the $t^3 g_1$ coupling is

$$\{T^3\,G_1\} = \frac{12}{\pi}\int_{\mathcal{A}} dxdy\,(\mathcal{C}_{t^3 g_1} + \mathcal{C}^*_{t^3 g_1}) = 1.6350\,. \tag{A.33}$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!}\{T^3G_1\}\,t^3 g_1 = 0.2725\, t^3 g_1$.

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